

## Supplementary Materials

# Computational Analysis of Novel Extended Lindley Progressively Censored Data

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Table S1: The Av.Es (1st column), RMSEs (2nd column) and MRABs (3rd column) of  $\theta$ .

$(n, m)$	Scheme	MLE			SE					GE			
					1					2			
Prior $\rightarrow$					2					1			
$\mu \rightarrow$										2			
										-2			
										+2			
(50,20)	1	1.5789	0.9271	0.9966	0.8932	0.1531	0.1619	0.9008	0.1064	0.1259	0.7026	0.1097	0.1218
					0.7419	0.1090	0.0590	0.7475	0.0542	0.0525	0.7101	0.1041	0.0899
					0.6533	0.1733	0.1845	0.6597	0.1409	0.1754	0.6324	0.1697	0.2095
	2	1.6131	0.9871	1.0805	0.8819	0.1461	0.1232	0.8898	0.0939	0.0898	0.7112	0.1036	0.0891
					0.6538	0.1743	0.1839	0.6604	0.1403	0.1744	0.6314	0.1710	0.2107
					0.8858	0.1493	0.1272	0.8937	0.0979	0.0937	0.7080	0.1061	0.0920
(50,40)	1	1.1243	0.8955	0.7100	0.7277	0.1172	0.0988	0.7332	0.0703	0.0835	0.8610	0.0868	0.0944
					0.7970	0.1017	0.0469	0.8022	0.0232	0.0208	0.7608	0.0860	0.0604
					0.7205	0.1209	0.1050	0.7260	0.0770	0.0925	0.6962	0.1145	0.1297
	2	1.1410	0.9281	0.7348	0.7984	0.1071	0.0477	0.8052	0.0240	0.0218	0.8336	0.0955	0.0888
					0.7151	0.1244	0.1098	0.7206	0.0819	0.0992	0.6907	0.1191	0.1367
					0.7398	0.1099	0.0608	0.8047	0.0228	0.0207	0.8367	0.0982	0.0914
(90,36)	1	1.1501	0.9121	0.7545	0.8661	0.1189	0.1219	0.8715	0.0738	0.0894	0.7309	0.0973	0.0863
					0.7956	0.1072	0.0485	0.8057	0.0241	0.0220	0.7502	0.0965	0.0690
					0.8683	0.1207	0.1246	0.8738	0.0760	0.0922	0.7353	0.0944	0.0809
	2	1.1541	0.9230	0.7660	0.7429	0.1083	0.0579	0.7485	0.0533	0.0515	0.7536	0.0925	0.0688
					0.8659	0.1193	0.1219	0.8714	0.0735	0.0892	0.7337	0.0930	0.0829
					0.7990	0.1067	0.0481	0.7454	0.0562	0.0546	0.7549	0.0921	0.0680
(90,72)	1	0.9572	0.5366	0.4737	0.7719	0.1132	0.0819	0.7794	0.0276	0.0257	0.8404	0.0642	0.0724
					0.7986	0.0940	0.0458	0.8025	0.0232	0.0206	0.7686	0.0686	0.0503
					0.7742	0.1112	0.0810	0.7815	0.0261	0.0231	0.7611	0.0849	0.0742
	2	0.9639	0.5523	0.4861	0.7983	0.1011	0.0465	0.8044	0.0225	0.0204	0.8423	0.0659	0.0596
					0.7722	0.1115	0.0819	0.7795	0.0269	0.0257	0.8399	0.0843	0.0719
					0.7982	0.1008	0.0463	0.8042	0.0223	0.0202	0.7613	0.0636	0.0590

Table S2: The Av.Es (1st column), RMSEs (2nd column) and MRABs (3rd column) of  $\sigma$ .

$(n, m)$	Scheme	MLE			SE					GE			
					1		2	1		2			
Prior $\rightarrow$													
$\mu \rightarrow$									-2				+2
(50,20)	1	0.4781	0.2088	0.3839	0.4991	0.1587	0.2831	0.5132	0.1178	0.2831	0.4661	0.0664	0.1653
					0.3899	0.1073	0.0573	0.4446	0.0580	0.0446	0.4239	0.0302	0.0287
	2	0.5021	0.2680	0.4799	0.4532	0.1323	0.1554	0.4667	0.0892	0.1667	0.3585	0.0534	0.1039
					0.4706	0.1252	0.0743	0.4802	0.0819	0.0802	0.4517	0.0422	0.0517
	3	0.6849	0.5546	1.0451	0.4538	0.1306	0.1604	0.4668	0.0882	0.1671	0.3534	0.0510	0.1164
					0.4685	0.1256	0.0732	0.4785	0.0814	0.0785	0.4493	0.0472	0.0493
(50,40)	1	0.4365	0.1337	0.2566	0.4523	0.1313	0.1568	0.4657	0.0809	0.1643	0.3672	0.0355	0.0821
					0.4224	0.0991	0.0504	0.4313	0.0516	0.0342	0.4056	0.0201	0.0162
	2	0.4417	0.1456	0.2773	0.3883	0.1051	0.1535	0.3999	0.0515	0.0921	0.3658	0.0367	0.0856
					0.4228	0.0985	0.0498	0.4316	0.0420	0.0340	0.4063	0.0200	0.0158
	3	0.4430	0.1946	0.3797	0.3867	0.1023	0.1555	0.3977	0.0514	0.0896	0.3648	0.0376	0.0880
					0.4233	0.0978	0.0491	0.4320	0.0403	0.0339	0.4071	0.0201	0.0156
(90,36)	1	0.4390	0.1398	0.2703	0.4085	0.1106	0.1534	0.4208	0.0491	0.0934	0.4306	0.0338	0.0766
					0.4374	0.0951	0.0491	0.4023	0.0406	0.0349	0.3713	0.0297	0.0239
	2	0.4524	0.1704	0.3242	0.4090	0.1111	0.1531	0.4214	0.0579	0.0928	0.4317	0.0347	0.0792
					0.4368	0.0957	0.0481	0.4441	0.0496	0.0441	0.4232	0.0295	0.0233
	3	0.5746	0.3953	0.7898	0.4087	0.1081	0.1531	0.4204	0.0574	0.0899	0.4327	0.0356	0.0818
					0.4361	0.0954	0.0471	0.4433	0.0479	0.0433	0.4226	0.0288	0.0226
(90,72)	1	0.4168	0.0943	0.1845	0.3904	0.1071	0.1433	0.4024	0.0433	0.0928	0.3845	0.0218	0.0470
					0.4196	0.0644	0.0305	0.4235	0.0318	0.0235	0.4116	0.0145	0.0116
	2	0.4193	0.1022	0.1998	0.3792	0.0911	0.1414	0.3884	0.0322	0.0704	0.3853	0.0209	0.0454
					0.4195	0.0652	0.0304	0.4236	0.0305	0.0236	0.4114	0.0144	0.0114
	3	0.4159	0.1465	0.2847	0.3743	0.0949	0.1405	0.3840	0.0349	0.0804	0.3860	0.0204	0.0446
					0.4188	0.0641	0.0298	0.4227	0.0314	0.0227	0.4109	0.0140	0.0109

Table S3: The Av.Es (1st column), RMSEs (2nd column) and MRABs (3rd column) of  $R(t)$ .

(n, m)	Scheme	MLE			SE					GE							
					1					1							
					2					2							
$\mu \rightarrow$										-2				+2			
(50,20)	1	0.9705	0.0121	0.0097	0.9632	0.0073	0.0029	0.9586	0.0027	0.0023	0.9567	0.0034	0.0027				
					0.9607	0.0070	0.0023	0.9612	0.0026	0.0018	0.9590	0.0033	0.0022				
	2	0.9697	0.0105	0.0087	0.9634	0.0074	0.0025	0.9606	0.0027	0.0019	0.9580	0.0035	0.0024				
					0.9600	0.0072	0.0023	0.9614	0.0026	0.0017	0.9634	0.0026	0.0018				
	3	0.9732	0.0103	0.0086	0.9603	0.0073	0.0024	0.9609	0.0027	0.0018	0.9582	0.0035	0.0024				
					0.9611	0.0069	0.0021	0.9616	0.0026	0.0017	0.9592	0.0034	0.0022				
(50,40)	1	0.9701	0.0086	0.0070	0.9626	0.0067	0.0024	0.9635	0.0024	0.0015	0.9609	0.0031	0.0020				
					0.9630	0.0065	0.0022	0.9631	0.0022	0.0015	0.9612	0.0028	0.0019				
	2	0.9701	0.0083	0.0068	0.9649	0.0061	0.0023	0.9635	0.0024	0.0015	0.9612	0.0031	0.0020				
					0.9630	0.0067	0.0021	0.9653	0.0020	0.0014	0.9656	0.0024	0.0018				
	3	0.9708	0.0081	0.0066	0.9630	0.0066	0.0021	0.9635	0.0023	0.0015	0.9612	0.0031	0.0020				
					0.9650	0.0060	0.0018	0.9654	0.0020	0.0014	0.9636	0.0025	0.0017				
(90,36)	1	0.9700	0.0095	0.0076	0.9582	0.0064	0.0025	0.9638	0.0024	0.0017	0.9610	0.0034	0.0022				
					0.9673	0.0064	0.0022	0.9626	0.0019	0.0014	0.9588	0.0032	0.0022				
	2	0.9698	0.0078	0.0065	0.9609	0.0070	0.0022	0.9631	0.0022	0.0014	0.9615	0.0023	0.0017				
					0.9626	0.0065	0.0021	0.9640	0.0023	0.0016	0.9612	0.0032	0.0022				
	3	0.9724	0.0076	0.0063	0.9635	0.0070	0.0023	0.9641	0.0023	0.0016	0.9615	0.0031	0.0021				
					0.9647	0.0053	0.0019	0.9626	0.0019	0.0015	0.9615	0.0023	0.0017				
(90,72)	1	0.9700	0.0067	0.0055	0.9648	0.0063	0.0021	0.9652	0.0021	0.0015	0.9632	0.0027	0.0019				
					0.9623	0.0048	0.0017	0.9678	0.0017	0.0014	0.9615	0.0023	0.0017				
	2	0.9700	0.0064	0.0054	0.9624	0.0049	0.0021	0.9626	0.0019	0.0015	0.9608	0.0029	0.0019				
					0.9642	0.0048	0.0017	0.9644	0.0016	0.0012	0.9633	0.0020	0.0014				
	3	0.9704	0.0062	0.0052	0.9625	0.0064	0.0020	0.9629	0.0022	0.0015	0.9608	0.0028	0.0019				
					0.9623	0.0049	0.0017	0.9650	0.0016	0.0011	0.9638	0.0020	0.0013				

Table S4: The Av.Es (1st column), RMSEs (2nd column) and MRABs (3rd column) of  $h(t)$ .

$(n, m)$	Scheme	MLE			SE					GE				
					1		2			1		2		
					Prior $\rightarrow$									
$\mu \rightarrow$														
(50,20)	1	0.4443	0.9910	0.6856	0.1680	0.0836	0.6451	0.1765	0.0722	0.6500	0.1402	0.0362	0.3430	
					0.0970	0.0258	0.0186	0.1185	0.0149	0.0141	0.0920	0.0125	0.0124	
					0.1368	0.0505	0.3225	0.1417	0.0461	0.3567	0.0673	0.0371	0.3550	
	2	0.5591	1.0879	0.8353	0.1473	0.0525	0.0431	0.1503	0.0394	0.0459	0.1380	0.0337	0.0336	
					0.1467	0.0520	0.3231	0.1415	0.0455	0.3553	0.0656	0.0388	0.3719	
					0.1368	0.0500	0.0426	0.1497	0.0391	0.0453	0.1374	0.0330	0.0329	
	3	0.6100	1.2889	0.9863	0.1362	0.0501	0.3191	0.0938	0.0137	0.1158	0.0802	0.0244	0.2317	
					0.1114	0.0225	0.0175	0.1132	0.0110	0.0088	0.1061	0.0065	0.0055	
					0.0885	0.0316	0.2488	0.0922	0.0148	0.1247	0.0788	0.0257	0.2449	
(50,40)	1	0.2236	0.3134	0.5588	0.1116	0.0225	0.0174	0.1134	0.0111	0.0090	0.1064	0.0065	0.0054	
					0.0873	0.0315	0.2529	0.0806	0.0247	0.2280	0.0778	0.0267	0.2548	
					0.1118	0.0223	0.0172	0.1136	0.0112	0.0091	0.1068	0.0065	0.0053	
	2	0.2382	0.5154	0.7057	0.0901	0.0312	0.2417	0.1410	0.0387	0.3506	0.1262	0.0223	0.2083	
					0.1175	0.0210	0.0172	0.0998	0.0085	0.0077	0.1142	0.0113	0.0098	
					0.0784	0.0361	0.2993	0.0820	0.0231	0.2144	0.1271	0.0232	0.2174	
	3	0.7190	0.5202	0.8808	0.1175	0.0209	0.0169	0.1186	0.0150	0.0142	0.1144	0.0114	0.0100	
					0.0767	0.0377	0.3167	0.0909	0.0157	0.1323	0.1272	0.0234	0.2183	
					0.1173	0.0207	0.0165	0.1183	0.0147	0.0139	0.1143	0.0112	0.0098	
(90,72)	1	0.1584	0.1572	0.5191	0.1022	0.0279	0.2095	0.1056	0.0078	0.0658	0.0919	0.0133	0.1198	
					0.1031	0.0092	0.0065	0.1035	0.0026	0.0021	0.1020	0.0033	0.0026	
					0.1028	0.0283	0.2096	0.1063	0.0082	0.0675	0.0925	0.0126	0.1140	
	2	0.1643	0.1720	0.5751	0.1032	0.0093	0.0066	0.1036	0.0027	0.0021	0.1021	0.0033	0.0026	
					0.1026	0.0282	0.2145	0.1061	0.0079	0.0658	0.0925	0.0127	0.1144	
					0.1025	0.0092	0.0066	0.1029	0.0029	0.0025	0.1014	0.0038	0.0031	
	3	0.1979	0.2500	0.5904										

Table S6: The ACLs ( $1^{st}$  column) and CPs ( $2^{nd}$  column) of ACI/HPD credible intervals of  $\sigma$ .

$(n, m)$ Prior $\rightarrow$	Scheme	ACI		HPD			
				1		2	
(50,25)	1	0.6953	0.912	0.3717	0.934	0.2833	0.956
	2	0.8236	0.906	0.3326	0.937	0.2805	0.957
	3	0.9467	0.896	0.3246	0.941	0.2625	0.960
(50,40)	1	0.4823	0.921	0.3316	0.936	0.2591	0.962
	2	0.5172	0.917	0.3127	0.945	0.2567	0.962
	3	0.7137	0.905	0.3084	0.948	0.2543	0.963
(90,36)	1	0.5092	0.918	0.3156	0.944	0.2356	0.965
	2	0.6100	0.914	0.3076	0.948	0.2354	0.965
	3	0.6118	0.912	0.3015	0.951	0.2329	0.966
(90,72)	1	0.3564	0.936	0.3131	0.947	0.1692	0.972
	2	0.3832	0.932	0.2835	0.955	0.1680	0.973
	3	0.5449	0.925	0.2824	0.955	0.1666	0.975

Table S7: The ACLs ( $1^{st}$  column) and CPs ( $2^{nd}$  column) of ACI/HPD credible intervals of  $R(t)$ .

$(n, m)$ Prior $\rightarrow$	Scheme	ACI		HPD			
				1		2	
(50,25)	1	0.0472	0.906	0.0161	0.948	0.0144	0.953
	2	0.0393	0.909	0.0159	0.948	0.0143	0.953
	3	0.0389	0.917	0.0154	0.949	0.0136	0.956
(50,40)	1	0.0341	0.922	0.0144	0.952	0.0139	0.955
	2	0.0326	0.924	0.0141	0.953	0.0138	0.955
	3	0.0317	0.926	0.0138	0.955	0.0129	0.957
(90,36)	1	0.0357	0.914	0.0143	0.952	0.0132	0.958
	2	0.0299	0.919	0.0135	0.955	0.0128	0.959
	3	0.0285	0.928	0.0133	0.954	0.0127	0.960
(90,72)	1	0.0255	0.931	0.0135	0.956	0.0101	0.965
	2	0.0245	0.932	0.0122	0.958	0.0102	0.965
	3	0.0235	0.934	0.0123	0.957	0.0101	0.965

Table S8: The ACLs ( $1^{st}$  column) and CPs ( $2^{nd}$  column) of ACI/HPD credible intervals of  $h(t)$ .

$(n, m)$ Prior $\rightarrow$	Scheme	ACI		HPD			
				1		2	
(50,25)	1	1.1204	0.907	0.1218	0.948	0.0762	0.974
	2	1.2551	0.899	0.1221	0.947	0.0927	0.968
	3	1.2883	0.895	0.1623	0.936	0.0934	0.967
(50,40)	1	0.8478	0.921	0.1000	0.955	0.0696	0.977
	2	0.9167	0.915	0.1016	0.953	0.0703	0.975
	3	1.1586	0.905	0.1204	0.949	0.0708	0.975
(90,36)	1	0.9191	0.913	0.0900	0.961	0.0559	0.981
	2	1.0598	0.911	0.0927	0.958	0.0565	0.979
	3	1.0918	0.908	0.0998	0.955	0.0567	0.979
(90,72)	1	0.4807	0.934	0.0856	0.968	0.0339	0.985
	2	0.5124	0.929	0.0861	0.967	0.0344	0.984
	3	0.7405	0.925	0.0941	0.960	0.0346	0.984

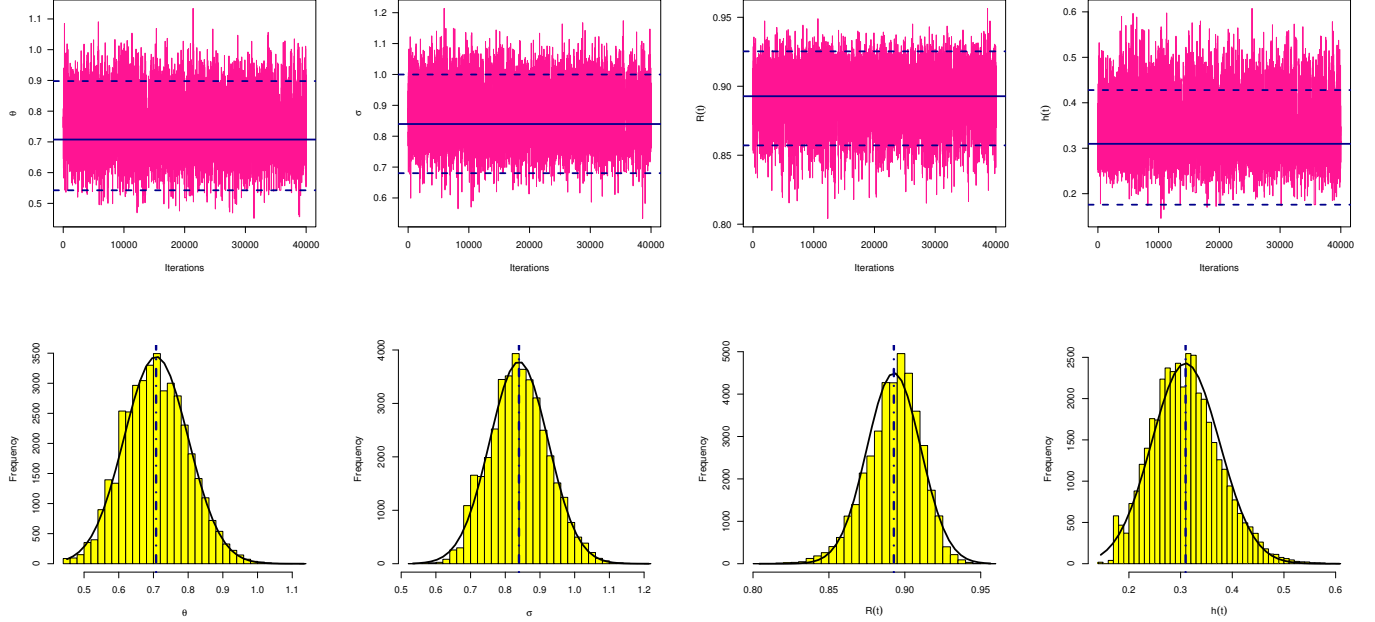


Figure S1: Trace (top) and Histograms (bottom) plots of  $\theta$ ,  $\sigma$ ,  $R(t)$  and  $h(t)$  under  $(0^9, 7, 7, 0^9)$  from vinyl chloride data.

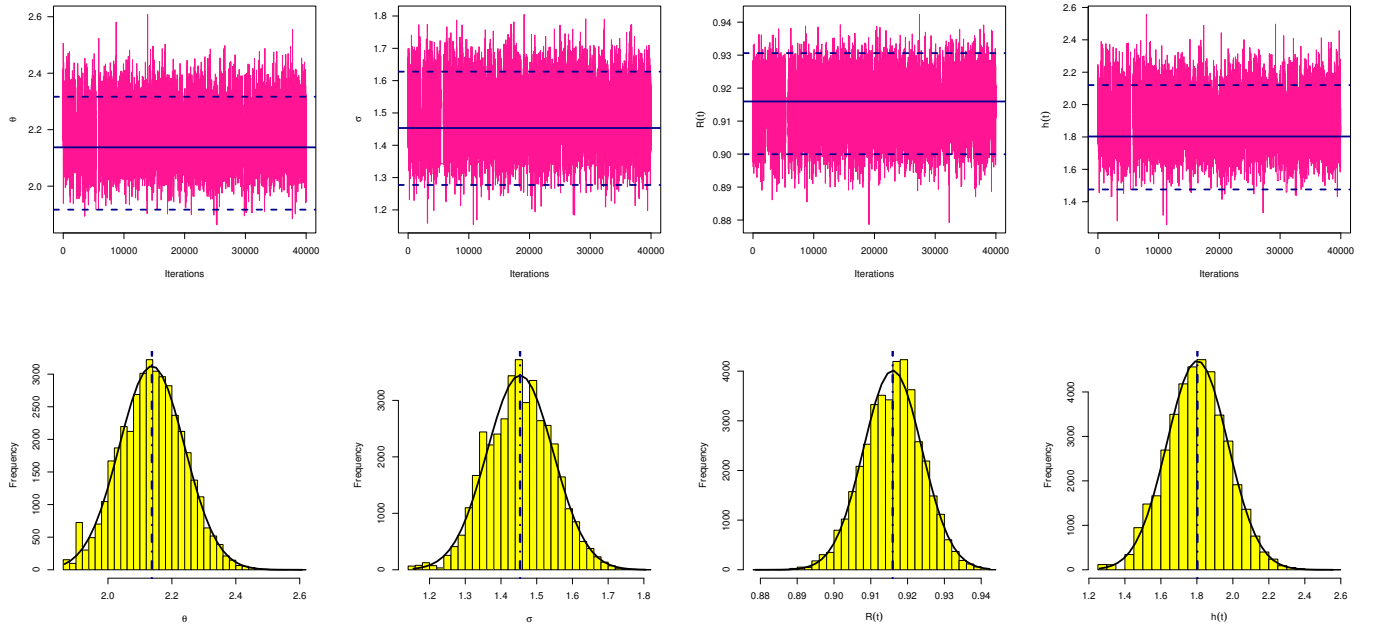


Figure S2: Trace (top) and Histograms (bottom) plots of  $\theta$ ,  $\sigma$ ,  $R(t)$  and  $h(t)$  under  $(0^{19}, 14)$  from vinyl chloride data.



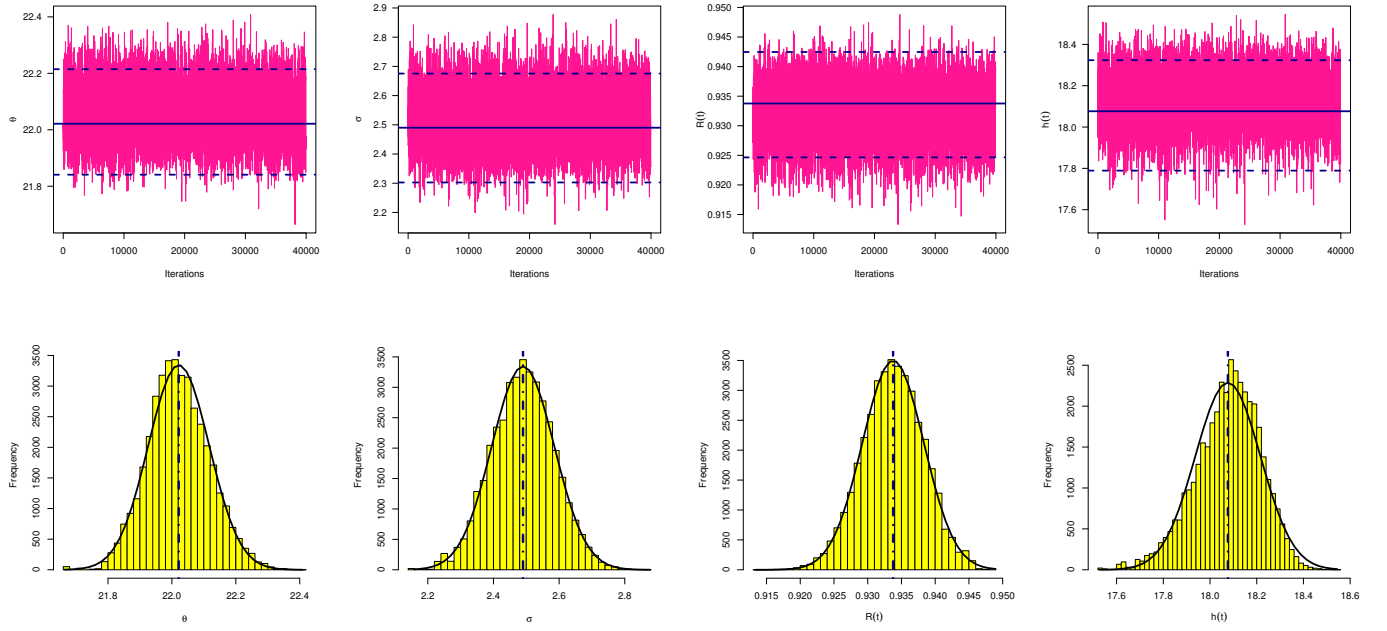


Figure S3: Trace (top) and Histograms (bottom) plots of  $\theta$ ,  $\sigma$ ,  $R(t)$  and  $h(t)$  under  $(0^4, 10, 10, 0^4)$  from RME data.

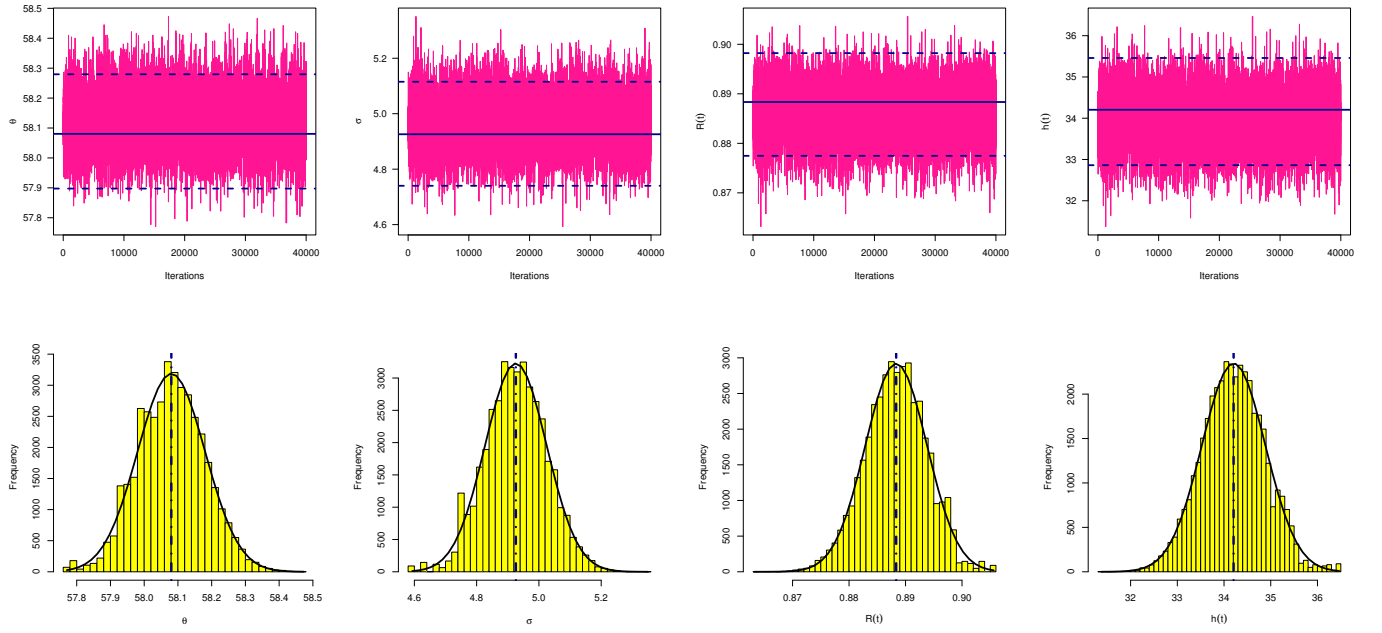


Figure S4: Trace (top) and Histograms (bottom) plots of  $\theta$ ,  $\sigma$ ,  $R(t)$  and  $h(t)$  under  $(0^9, 20)$  from RME data.